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TIME-DOMAIN METHOD FOR COMPUTING
FORCES AND MOMENTS ACTING ON
THREE DIMENSIONAL SURFACE-PIERCING
SHIP HULLS WITH FORWARD SPEED

by

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are used to advance the flow. This time-domain computation can
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compute the added mass and damping forces and moments in the
frequency domain. Wave-excited forces can be computed by a simi-
lar method. As a check on the computer program, the added mass
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hull and free surface.

INTRODUCTION

This report describes a computational method designed to simulate arbitrary linearized motions of a surface-piercing three-dimensional ship hull in the time domain. The hull is specified by a set of quadralateral surface panels describing the hull portion directly beneath the static waterline. The numerical method calculates the forces and moments generated by arbitrary motions of the body and/or the action of an ambient wave field acting on the body under the assumption that the flow velocities can be linearized.

The method may be classified as a time-domain computation analogous to the frequency-domain methods developed by Chang (1) for three-dimensional ship hulls. The time-domain method is the more fundamentally numerical approach to this class of problems. The time-domain method does not use Green's functions representing solutions of the free-surface equations for singularities following prescribed conditions (i.e., moving with uniform speed and oscillating as in reference [1]). Thus the time domain method provides the flexibility of arbitrary motions. It can easily be generalized to include a time-dependent hull shape or other generalized features.

Limitations on computer times and storage do not, however, allow this "brute force" type method to serve as a practical tool in three dimensions at the present time. With continued improvements in computer technology, however, methods of this nature may, in the near future, be explored for practical applications. It is hoped that the techniques described here will provide a framework for later work of this nature.

A basic objective in the development of the computational method was to establish the method in as simple a form as possible. The resulting numerical technique described in following sections of this report, is a generalization of the two-dimensional method applied in reference [2], expanded to three dimensions with some

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improvements in the numerical method. Another objective was to examine the possible problem areas such as the free surface area interior to the hull and pressure computations. These areas are also discussed in this report. Direct application to linearized seakeeping computations is discussed. Finally a sample problem - heave oscillations of a semisubmerged sphere - is presented.

Problem Formulation

Consider a coordinate system (x, y, z, t) with z positive downward and z = 0 at the static waterline. Far from the origin the flow is at rest. This space-fixed coordinate system is the basis for describing the fluid flow in general and the free surface in particular. The velocity potential $\phi(\vec{x}, t)$ satisfies Laplaces equation throughout the fluid.

$$\nabla^2 \phi = 0 \quad z > 0$$

The free surface elevation field $\eta(x, y, t)$ and its time derivative $\dot{\eta}(x, y, t)$ are defined over the entire plane z = 0. Their values at any time provide sufficient initial conditions to define the free surface problem. Exterior to the intersection of the hull surface the elevation field satisfies the dynamic and kinematic free surface conditions

$$\frac{\partial \phi}{\partial t} = -g\eta$$

and

$$\frac{\partial \phi}{\partial z} = -\frac{\partial \eta}{\partial t}$$

On the free surfaces interior to the hull any condition could be applied. In this case, however, we assume that the above linearized equations apply over the entire z = 0 plane. An arbitrary surface-piercing hull is defined by the surface,

$$F(x, y, z, t) = 0 \quad z > 0 \; ,$$

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with the exciting normal velocity v_n induced by body motions or interactions with the ambient wave field prescribed on the surface of the hull,

$$v_n(\vec{x},\ t) = \vec{v}*(\vec{x},\ t)\cdot\vec{n},\ \vec{x}\ \varepsilon\ S_{hull}\ .$$

(In practice the normal acceleration is specified rather than the velocity.) The disturbance generated by the body motion is divided into two parts

$$\phi = \phi_{BD} + \phi_{FS}$$

where ϕ_{BD} represents the instantaneous effect of the body, while ϕ_{FS} represents the free surface disturbance generated by the body over all previous motions or interactions with the ambient wave field. In particular the boundary conditions at the free surface $(z = 0)$ are

$$\phi_{BD} = 0 \qquad z = 0$$

$$\frac{\partial \phi_{FS}}{\partial t} = -g\eta \qquad z = 0$$

and on the body,

$$\frac{\partial}{\partial n}\ (\phi_{BD} + \phi_{FS}) = \vec{v}*\cdot\vec{n} \qquad \vec{x}\ \varepsilon\ S_B$$

It is assumed that body components velocities are small, i.e.,

$$\vec{v}_{BD}(\vec{x}) = 0\ (\varepsilon)$$

and

$$\vec{v}_{FS}(x) = 0\ (\varepsilon)\ .$$

Any terms of order ε^2 are dropped. For example, this analysis applies to the problem of ship motion in a seaway at finite speed if it can be assumed that the flow field induced by forward motion alone and the flow field induced by the seaway acting on the ship are both small and do not interact. In this case of finite forward speed, the velocity at any point on the hull $\vec{x}$ is

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$$\vec{v}^S(\vec{x}) = U \, \vec{e}_x + O\,(\varepsilon)$$

where U is the forward speed. The disturbance and resulting forces generated by forward speed alone are steady (for steady u) and are not addressed here (although this method could be easily applied). It is assumed that the velocity field generated by forward speed alone is of order ε.

Outline of Numerical Method

Starting from a condition of rest, the simulation is achieved by a series of small time increments. At each time step, three fundamental sets of variables - which essentially define the flow at the instant - are computed;

(a) The time rate of change of the strengths of the hull source panels in a hull-fixed system are computed from the hull acceleration boundary condition. This determines the time-derivative of the body-induced component of the flow, $\dot{\phi}_{BD}$ (as well as ϕ_{BD} through integration).

(b) The free surface elevation field $\eta(x, y)$ and its time derivative $\dot{\eta}(x, y)$ are computed by advancing the free surface by a small increment in time and adding the wave elevations generated by the hull source panels over that increment. This defines the free surface component of the flow, ϕ_{FS} and its time derivative in the space-fixed system.

(c) Pressures acting on the hull are computed separately for the free surface and body components: p_{FS}, p_{BD}. Both are computed at hull panel center points and added to yield total forces and moments.

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Computation of Panel Source Strengths

The hull is represented numerically by a set of N_B quadralateral panels. Each panel is specified by the coordinates of the four corner points, a panel center point, and a normal vector. The fluid velocity at each center point in the normal direction may be written as

$$\vec{v}_{BD_i} \cdot \vec{n}_i + \vec{v}_{FS_i} \cdot \vec{n}_i = \vec{v}_i^*(t) \cdot \vec{n}_i \qquad i = 1, 2, \dots N_B$$

where $\vec{v}_i^*(t)$ is a prescribed forcing function depending on the motion of the hull and the ambient wave field. Since the free surface induced velocity $\vec{v}_{FS}$ is assumed to be available from the free surface representation, this equation fixes $\vec{v}_{BD_i} \cdot \vec{n}_i$ at the center point of each hull panel.

A simple source and its image is assumed to be distributed over each hull panel

$$G(x, y, z, x', y', z') = ((x-x')^2 + (y-y')^2 + (z-z')^2)^{-1/2}$$
$$- ((x-x')^2 + (y-y')^2 + (z+z')^2)^{-1/2}$$

with

$$\phi_{BD}(x, y, z) = \sum_{i=1}^{N_B} \sigma_i \oiint_{S_i'} G(x, y, z, x', y', z')dS' ,$$

where σ_i is the strength on the ith source panel and S_i' represents the panel surface.

Consider a simple source of unit strength acting over a plane surfaces',

$$\phi(\overline{x}) = \oiint dS' \frac{1}{|\vec{x}' - \vec{x}|}$$

Then it may be shown analytically that the velocity normal to the plane is identical to the solid angle occupied by the panel surface

and the velocity tangential to the plane surface can be written as a line integral around the boundary of the panel. Numerical application of these results produces an array E_{ij} giving the normal velocity component at the center point of panel i induced by a uniform source distributed over panel j and its image,

$$\vec{v}_{BD_i} \cdot \vec{n}_i = \sum_{d=1}^{N_B} E_{ij} \sigma_j \qquad i = 1, \ldots N_B.$$

This provides a set of linearized equations which can be used to determine the source strengths σ_j. In practice, it was found that better estimates for body-induced pressures could be obtained by using the local acceleration rather than the velocity directly. The time derivative $\dot{\sigma}_j$ relative to a hull fixed coordinate system is computed at the beginning of each time step from

$$\sum_{d=1}^{N_B} E_{ij} \dot{\sigma}_j = \vec{a}_{BD} \cdot \vec{n}_i = a_i^*(t) - \vec{a}_{FS_i} \cdot \vec{n}_i - U \frac{\partial}{\partial x} \vec{v}_{FS_i} \cdot \vec{n}_i$$

where $\vec{a}_{FS_i}$ and $\vec{v}_{FS_i}$ are the free-surface induced acceleration and velocity (in space-fixed coordinates) at point i and $a_i^*(t)$ is the prescribed normal acceleration in hull-fixed coordinates. For example, if the body has surge acceleration $a_1(t)$, then $a_i^*(t) = a_1(t) \cdot n_{x1}$. The source strengths at the center can be computed directly from the local velocities if desired. In practice it was found that they could be computed adequately by integrating their derivatives in time.

$$\sigma_i(t_n + \Delta t) \sim \sigma_i(t_n) + \Delta t \, \dot{\sigma}_i(t) .$$

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Representation of Wave Elevation Field

The representation of the wave elevation field and its time derivative by their spectral components allows the linearized free surface boundary conditions to be applied easily when adapting the free surface component of the flow. It also aids in computing free surface induced pressures and velocities on the hull. An arbitrary real continuous function $\eta(x, y)$ defined over the $z = 0$ plane can be written in the form

$$\eta(x, y) = \int_0^\infty dkx \int_{-\infty}^\infty dky \, A(kx, ky) \, e^{i(kx \cdot x + ky \cdot y)}$$

where $A(kx, ky)$ is complex function and the elevation $\eta(x, y)$ is taken to be the real part of the expression on the right hand side. Numerically $\eta(x, y)$ can be represented by

$$\eta(x, y) = \sum_{n=1}^{NKX} \Delta kx_n \sum_{m=1}^{NKY} \Delta ky_m \left[A_{nm}(t) \, e^{i(kx_n \cdot x + ky_m \cdot y)} + A^*_{nm}(t) \, e^{i(kx_n \cdot x - ky_m \cdot y)} \right]$$

Similarly if $\dot{\eta}(x, y)$ is defined as the time derivative of $\eta(x, y)$ due to free surface effects alone, that is the rate of change of $\eta(x, y)$ in the absence of the hull, it may be represented numerically by

$$\dot{\eta}(x, y) = \sum_{n=1}^{NKX} \Delta kx_n \sum_{m=1}^{NKY} \Delta ky_m \left[B_{nm}(t) \, \sigma_{nm} \, e^{i(kx_n \cdot x + ky_m \cdot y)} + B^*_{nm}(t) \, \sigma_{nm} \, e^{i(kx_n \cdot x - ky_m \cdot y)} \right]$$

where $\sigma_{nm} > 0$ is defined by

$$\sigma^2_{nm} = g(kx_n^2 + ky_m^2)^{1/2}$$

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Thus in the absence of the hull the time derivative of A_{nm} is given by

$$\frac{d}{dt} A_{nm}(t) = B_{nm}(t) \sigma_{nm}$$

and

$$\frac{d}{dt} A^*_{nm}(t) = B^*_{nm}(t) \sigma_{nm}$$

From the kinematic free surface condition for the free surface component alone,

$$\frac{\partial \eta}{\partial t} = - \frac{\partial \phi_{FS}}{\partial z} \quad , \quad z = 0$$

a special representation for the free surface potential may be derived

$$\phi_{FS}(x, y, z) = \sum_{n=1} \Delta kx_n \sum_{m=1} \Delta ky_m \left[B_{nm}(t)\sigma_{nm} k^{-1}_{nm} e^{i(kx_n \cdot x + ky_m \cdot y)} \right.$$

$$\left. + B^*_{nm}(t) \sigma_{nm} k^{-1}_{nm} e^{i(kx_n \cdot x - ky_m \cdot y)} \right] e^{-k_{nm} \cdot z}$$

where

$$k^2_{nm} = kx^2_n + ky^2_m$$

Now the dynamic free surface condition,

$$\frac{\partial \phi_{FS}}{\partial t} = -g\eta \quad z = 0,$$

can be applied to yield the remaining equation to be used when updating the free surface in time,

$$\frac{d}{dt} B_{nm}(t) = -A_{nm}(t) \sigma_{nm}$$

$$\frac{d}{dt} B^*_{nm}(t) = -A^*_{nm}(t) \sigma_{nm}$$

The free surface induced pressure and normal component may be
computed from A_{nm} and A^*_{nm} directly

$$p_{FS} = -\rho \frac{\partial \phi_{FS}}{\partial t}$$

$$= \rho g \sum_{n=1}^{NKX} \Delta ky_n \sum_{m=1}^{NKY} \Delta ky_m \ e^{(ikx_n \cdot x - k_{nm} \cdot z)} \left[A_{nm}(t) \ e^{iky_m \cdot y} + A^*_{nm}(t) \ e^{-iky_m \cdot y} \right]$$

and

$$\vec{a}_{FS} \cdot \vec{n} = -g \sum_{n=1}^{NKX} \Delta kx_n \sum_{m=1}^{NKY} \Delta ky_m \ e^{(ikx_n \cdot x - k_{nm} \cdot z)}$$

$$\left[A_{nm}(t) \cdot e^{iky_m \cdot y} (iky_n \cdot n_x + iky_m \cdot n_y - k_{nm} \cdot n_z) \right.$$

$$\left. + A^*_{nm}(t) \cdot e^{-iky_m \cdot y} \cdot (ikx_n \cdot n_x - iky_m \cdot y - k_{nm} \cdot n_z) \right]$$

The distribution of values of kx_n and ky_m where A_{nm}, A^*_{nm}
and B_{nm}, B^*_{nm} are specified is critical. For practical computing
times, the number of points should be kept to an absolute minimum.
On the other hand, the free surface representation must be valid

(1) Over the hull surface throughout the flow in the immediate
     neighborhood of the hull surface and,

(2) Over the maximum anticipated time interval of the
     simulation.

The hull and the free surface flow is assumed to be
located in the general neighborhood of the origin of the (x, y)
plane. Suppose the free-surface component of the flow were periodic
in space with periodic lengths $2\pi \cdot Lx$ and $2\pi \cdot Ly$ in the x and y
directions respectively. Then the continuum of kx and ky are
replaced by the discrete values -

$$kx_n = n/L_x \qquad n = 1, 2, 3 \ldots$$

and

$$ky_m = m/L_y \qquad m = 1, 2, 3 \ldots$$

Similarly, if the free surface component of flow were periodic in time with a period of $2\pi T$, then

$$k = (kx^2 + ky^2)^{1/2} = n^2/gT^2$$

These results are applied in obtaining a kx, ky distribution satisfying the following requirements. The free surface flow representation must be valid.

(1) Over a region extending L_x in the x direction and L_y in the y direction from the origin of the (x, y) plane,

(2) Over features with characteristic lengths of ℓ_x and ℓ_y or larger

(3) Over time intervals of length T or shorter

without significant influence from the discretization of kx and ky.

Condition (1) imposes maximum step sizes for kx_n and ky_m

$$kx_{n+1} - kx_n \leq L_x^{-1}$$

$$ky_{m+1} - ky_m \leq L_y^{-1} .$$

Condition (2) sets the minimum upper bounds for kx_n and ky_m

$$kx_{NKX} \geq \ell_x^{-1}$$

$$ky_{MKY} \geq \ell_y^{-1} .$$

Condition (3) cannot be applied directly for independent values of kx_n and ky_m. It can be adequately replaced by a pair of similar conditions,

$$\sqrt{kx_{n+1}} - \sqrt{kx_n} \leq (T \cdot \sqrt{g})^{-1}$$

and

$$\sqrt{ky_{m+1}} - \sqrt{ky_m} \leq (T \cdot \sqrt{g})^{-1}$$

These final conditions usually affect only the few lowest values of kx and ky, depending on the values of the other parameters.

Advancement of the Free Surface in Time

With each time step, the free surface portion of the flow as represented by $A_{nm}(t)$, $A^*_{nm}(t)$ and $B_{nm}(t)$, $B^*_{nm}(t)$ (i.e., $\eta(x, y)$) must be advanced. The new values are those which would result if the hull were not present over the time interval (from t_n to $t_n + \Delta t$) added to estimates for the free surface flow generated by the body (i.e., by the source panels and their images) over the small time interval. The algorthms used to update the coefficients are

$$A_{nm}(t_n + \Delta t) = A_{nm}(t_n) \cos\sigma_{nm} \Delta t + B_{nm}(t_n) \sin\sigma_{nm} \Delta t + \Delta A^{BODY}_{nm}$$

and

$$B_{nm}(t_n + \Delta t) = B_{nm}(t_n) \cos\sigma_{nm} \Delta t - A_{nm}(t_n) \sin\sigma_{nm} \Delta t + \Delta B^{BODY}_{nm}$$

(with identical algorthms for updating A^*_{nm} and B^*_{nm}). Since the body-induced potential is defined as zero on the free surface,

$$\phi_{BD}(x, y, z) = 0, \quad z = 0,$$

the influence of the body source panels and their images on B_{nm}, B^*_{nm} over this short time period are second order in Δt

$$\Delta B^{BODY}_{nm}, \quad \Delta B^{BODY*}_{nm} \equiv \mathcal{O}(\Delta t)^2$$

The free surface elevation induced by the body source panels and their images over a small time interval Δt may be computed by summing the contributions of each panel and its image. A panel with uniform density σ_i acting over a surface of area A_i is replaced by a single point source with strength

$$s_i(t) = A_i(\sigma_i(t_n) + (t-t_n) \dot{\sigma}(t_n))$$

located at the panel center point (x_i, y_i, z_i) as defined at the midpoint of the interval, $t = t_n + 1/2 \Delta t$.

The vertical velocity at $z = 0$ induced by these source points and their images may be written as

$$\frac{\partial \phi_B(t)}{\partial z} = - \sum_{i=1}^{N_B} \frac{2z_i \, s_i(t)}{\left|(x-x_i)^2 + (y-y_i)^2 + z_i^2\right|^{3/2}} \; .$$

This can be expressed in integral form as

$$\Delta \eta_B(x,y) = \frac{2}{\pi} \int_{t_n}^{t_n + \Delta t} dt \sum_{i=1}^{N_B} s_i(t) \int_0^\infty dkx \; e^{ikx(x-x_i)} \int_{-\infty}^\infty dky \; \cos ky(y-y_i)$$

$$\cdot \; e^{-(kx^2+ky^2)z_i}$$

Thus, to second order in Δt, ΔA_{nm}^{BODY} and ΔA_{nm}^{BODY*} can be written as

$$\Delta A_{nm}^{BODY} = 2 \frac{\Delta t}{\pi} \sum_{i=1}^{N_B} s_i(t_n + \frac{1}{2} \Delta t) \; e^{-i(kx_n \cdot x_i + ky_m \cdot y_i)} \; e^{-k_{nm} z_i}$$

and

$$\Delta A_{nm}^{BODY*} = 2 \frac{\Delta t}{\pi} \sum_{i=1}^{N_B} s_i(t_n + \frac{1}{2} \Delta t) \; e^{-i(ky_n \cdot x_i - ky_m \cdot y_i)} \; e^{-k_{nm} z_i}$$

Also in second order in Δt, the changes in B_{nm} induced by the body is

$$\Delta B_{nm}^{BODY} = - \frac{1}{2} \frac{\partial}{\partial t} A^{BODY}(t) \; \sigma_{nm}(\Delta t)^2$$

or

$$\Delta B_{nm}^{BODY} = - \frac{(\Delta t)^2}{\pi} \sum_{i=1}^{N_B} s_i(t_n) \; e^{-i(kx_n \cdot x_i + ky_m \cdot y_i)} \; e^{-k_{nm} z_i} \sigma_{nm}$$

with a similar expression for ΔB_{nm}^{BODY*}.

Computation of the Body-Induced Pressures

The total force acting on the hull surface is determined
from the pressures computed at the center points of the hull panels.
These pressures are assumed to act uniformly over the panel sur-
face. From Bernoulli's equation, the pressure at a point fixed in
space is

$$p = -\rho \frac{\partial \phi}{\partial t} + 0 \ (\epsilon^2)$$

where the time derivative is in a space-fixed coordinate system
and the velocity field relative to this space-fixed system is
assumed to be of order ϵ. The pressure may be written as the sum
of free-surface and body-induced components

$$p = p_{FS} + p_{BD}$$

where

$$p_{FS} = -\rho \frac{\partial \phi_{FS}}{\partial t}$$

and

$$p_{BD} = -\rho \frac{d\phi_{BD}}{dt} + \rho \ \vec{v}_s \cdot \vec{\nabla} \ \phi_{BD}$$

where $\vec{v}_s$ is the velocity of a point on the hull surface. If the
body moves with translational velocity $\vec{v}_B$ and rotational velocity
$\vec{\omega}_B$ then

$$\vec{v}_s = \vec{v}_B + \vec{\omega}_B \times \vec{x} \ .$$

The term $\frac{d\phi_{BD}}{dt}$ represents the time derivative of the
potential at a point fixed on the hull (i.e. at a panel center).
In most cases of interest

$$\vec{v}_s = U \ \vec{e}_x + 0 \ (\epsilon)$$

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where U is the forward speed and $\vec{e}_x$ is the unit vector in the x
direction. Unless otherwise specified the body-induced pressure
is assumed to be

$$p_{BD} = -\rho \frac{d\phi_{BD}}{dt} + \rho U \frac{\partial\phi_{BD}}{\partial x} .$$

The time derivative $\dfrac{d\phi_{BD_i}}{dt}$ at the center point of panel i can be
calculated from the previously computed time derivative of the
source strengths in a hull-fixed system,

$$\frac{d\phi_{BD_i}(t_n)}{dt} = \sum_{j=1}^{N_B} P_{ij} \, \dot{\sigma}_j(t_n)$$

where P_{ij} represents the potential at the center of panel i induced
by a uniform source density of unit strength acting over panel j.
Unfortunately, elements of the matrix P_{ij} giving the potential are
not as easy to compute numerically as the matrix E_{ij} giving the
velocity. At present integrals of the form

$$P_{ij} = \iint\limits_{S_j} dS \left(\frac{1}{|\vec{x}_i - \vec{x}|} - \frac{1}{|\vec{x}_i - \vec{x}^I|} \right)$$

where $\vec{x}^I$ is the image of point $\vec{x}$ on surface S_j, are estimated by
dividing the surface S_j into many small elements and integrating
numerically with the integrand evaluated at the center of each
subelement. This 'brute force' method can consume significant
computing time if resonable accuracy is desired. These elements
need be computed only once, however, if the hull does not change
over the time period of simulation.

The forces and moments acting on the body in a hull-
fixed coordinate system may be written in the form,

$$F_m = - \sum_{i=1}^{N_B} n_{mi} A_i p_i \qquad m = 1 \ldots 6$$

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where F_m is a generalized force corresponding to the mth degree of freedom, n_{mi} is the generalized normal, A_i is the panel area and p_i is the pressure on the ith panel. From this form, the force F_m' resulting from the $\frac{d\phi_{BD}}{dt}$ term can be written as

$$F_m' = \rho \sum_{i=1}^{N_B} n_{mi} A_i \frac{d}{dt} \phi_{BD_i}$$

$$= \rho \sum_{i=1}^{N_B} n_{mi} A_i \sum_{j=1}^{N_B} P_{ij} \dot{\sigma}_j$$

$$= \rho \sum_{j=1}^{N_B} \underline{PP}_{mj} \dot{\sigma}_j \quad .$$

The $\underline{PP}_{mj}$ array of $(6 \times N_B)$ elements can be evaluated once at the beginning of the computation. $(t = t_n)$.

Similarly the methods used to calculate the E_{ij} matrix can also give elements of a matrix X_{ij} such that

$$\frac{\partial}{\partial x} \phi_{BD_i} (t_n) = \sum_{j=1}^{N_B} X_{ij} \sigma_j (t_n) \quad .$$

The forces resulting from the second term of the equation for p_{BD} are

$$F_m'' = -\rho U \sum_{j=1}^{N_B} n_{mi} A_i \sum_{j=1}^{N_B} X_{ij} \sigma_j (t_n)$$

$$= -\rho U \sum_{j=1}^{N_B} \underline{PX}_{mj} \sigma_j$$

where $\underline{PX}_{mj}$ is a $(6 \times NB)$ matrix similar to $\underline{PP}_{mj}$.

Alternative Techniques

Certain subelements of the computation can be represented by alternative forms. The computational method described above was chosen from the following set of possibilities

(1) Either source or dipole panels can be used to represent the hull surface. If dipoles are used, then the normal velocity on the image panel is the negative of the velocity on the panel itself so that the net mass inflow into the region enclosed by the hull and its image is zero. The major numerical <u>advantage</u> of dipole panels is that the potential induced by a uniform dipole strength of unit magnitude is easily computed at any arbitrary point in space (it is simply the solid angle enclosed by the panel relative to that point). This allows one to compute the pressures induced by the time derivative of the body-induced potential much more easily numerically than if source panels were used. The primary <u>disadvantage</u> of dipole panels is that the free surface disturbance generated by them is discontinuous along points where the hull intersects the free surface. That is, there is a jump in elevation between the interior and exterior regions. This requires much finer resolation in the representation of the free-surface and, therefore, more points in k-space for the spectral distribution of the free-surface.

(2) Alternative conditions can be applied to the free surface interior to the hull. For example instead of using the linearized free-surface condition after the entire free surface, the surface elevation interior to the hull can be constrained to be uniformly zero by placing surface panels over this region. One possible advantage of "adding a lid" is that it suppresses standing waves which otherwide are excited in the hull interior. The major disadvantage of a "lid" is the extra computing time

required. In theory these internal standing waves produce no net pressure on the (exterior) hull surface. That is the body-induced and free-surface induced components of pressures cancel at points exterior to the hull surface. In practice, they may not cancel unless the pressures are evaluated numerically in certain ways. It has been found that using the free surface induced acceleration, rather than velocity, in the hull normal boundary condition is very effective in removing any influence on the pressures from the standing waves generated internal to the hull.

(3) The spectral representation of the free surface can use either a rectangular or cylindrical representation in spectral space. That is $\eta(x, y)$, the elevation, can be represented either as

$$\eta(x,y) = \sum_{n=1}^{NKX} \Delta kx_n \sum_{m=1}^{NKY} \Delta ky_m \ e^{ikx_m \cdot x} \cdot \left[A_{nm} \ e^{iky_m \cdot y} + A^*_{nm} \ e^{-iky_m \cdot y} \right]$$

or

$$\eta(x,y) = \sum_{n=1}^{NK} k_n \cdot \Delta k_n \sum_{m=1}^{NTH} \Delta\theta \ A_{nm} \ e^{ik_n(x \cos\theta_m + y \sin\theta_m)}.$$

$$\left(\theta_1 = -\frac{\pi}{2}, \ \theta_{NTH} = +\frac{\pi}{2} \right)$$

Both methods have been programmed. They are very similar. The advantage of the rectangular system is that the length scales of the free surface are independent in the x and y directions. This is particularly useful for typical ship hulls where the hull length is much greater than the transverse dimensions of the ship and the flow is expected to change much more slowly in the x direction than in the y direction.

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Application to Linearized Frequency-Domain Seakeeping Characteristics with Forward Speed

Consider a hull moving with forward speed U and oscillating about its mean position defined by the surface

$$F(x - Ut, y, z) = 0$$

The flow velocity perturbation induced by the forward motion alone is assumed to be small, i.e. $O(\varepsilon)$. Harmonic perturbations about the mean flow field are also assumed to be of order ε. Interactions between the steady and harmonic portions of the flow field are assumed to be of higher order in ε.

Motions are described in a coordinate system (x^*, y^*, z^*) moving with the mean hull, i.e. with forward speed U. The coordinates of this moving system relative to that of the space-fixed (x, y, z) system are given by

$$x^* = x - Ut$$
$$y^* = y$$
$$z^* = z \ .$$

The velocities of the craft relative to a moving (x^*, y^*, z^*) ship-fixed system are written for the six degrees of freedom as

v_1 = surge velocity (+ forward)

v_2 = sway velocity (+ to starboard)

v_3 = heave velocity (+ downward)

v_4 = roll velocity (+ for right-hand rotations about x axis)

v_5 = pitch velocity (+ for right-hand rotations about y axis)

v_6 = yaw velocity (+ for right-hand rotations about z axis)

All velocities are assumed to be of order ε and harmonic in time with frequency ω. [Since these velocities are specified in a

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ship-fixed system, they are related to both velocities and angular displacements in a space-fixed system for finite forward speed. For example,

$$v_3(t)_{SHIP} = v_3(t)_{SPACE} - \theta_5(t) \cdot U$$

where U is the forward speed and θ_5 is the pitch angle.]

The normal velocity at a point $\vec{x}*$ on the surface is given by harmonic velocities in a ship-fixed system with magnitude v_m and phase θ_m is

$$\vec{v}* \cdot \vec{n} = \sum_{m=1}^{6} n_m(\vec{x}*) \; v_m \; \cos(\omega t + \theta_m)$$

where $n_4(\vec{x}*)$, $n_5(\vec{x}*)$, and $n_6(\vec{x}*)$ are generalized normals defined as

$$n_4(\vec{x}*) = n_2(\vec{x}*) \cdot x_3^* - n_3(\vec{x}*) \cdot x_2^*$$

$$n_5(\vec{x}*) = n_3(\vec{x}*) \cdot x_1^* - n_1(\vec{x}*) \cdot x_3^*$$

$$n_6(\vec{x}*) = n_1(\vec{x}*) \cdot x_1^* - n_2(\vec{x}*) \cdot x_1^*$$

similarly the normal acceleration produced by the motion is

$$\vec{a}* \cdot \vec{n} = - \sum_{m=1}^{6} n_m(\vec{x}*) \; v_m \; \omega \; \sin(\omega t + \theta_m)$$

The problem is to compute the forces and moments induced by these harmonic motions for arbitrary ω from a set of time-domain solutions (one for each of the six degrees of freedom) for impulsive motions. Since the problem is linearized, a single degree of freedom can be considered without loss of generality. Consider, for example, the steady harmonic motion described by

$$\vec{v}* \cdot \vec{n} = v_j \; n_j(\vec{x}*) \; \cos\omega t$$

$$\vec{a}* \quad \vec{n} = -\omega \; v_j \; n_j(\vec{x}*) \; \sin\omega t$$

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Assume that the time-domain solution based on the computational method described above is available for the following problem in the limiting case as $\gamma \to 0$. The flow is at rest for $t < 0$. Starting at time $t = 0$ the flow is disturbed by motion in the jth degree of freedom described by the body boundary condition,

$$\vec{v}^* \cdot \vec{n} = n_j(\vec{x}^*) \qquad 0 \le t$$

$$= 0 \qquad t < 0$$

i.e., $\qquad \vec{v}^* \cdot \vec{n} = n_j(\vec{x}^*) \, H(t)$

where $H(t)$ is the unit step function and where $\vec{x}^*$ is any point on the hull surface. Define the resulting potential at a point x^* on the hull as

$$\tilde{\phi}_j(\vec{x}^*, t) \qquad t > 0$$

and the forces and moments relative to the (x^*, y^*, z^*) system in the six degrees of freedom as

$$\tilde{F}_{kj}(t) \qquad k = 1, 2, 3 \dots 6 \qquad t > 0$$

The tilda symbol $\sim$ indicates a time-domain solution obtained from the numerical simulation. The j subscript refers to the degree of freedom of the motion. The k subscript refers to the degree of freedom of the force or moment.

The forward speed U should be included in the time-domain computations. Although forward speed does not appear directly in the body boundary condition, it enters the computation in two ways. First the mean hull surface moves

$$F(x - Ut, y, z) = 0$$

with speed U so that over the time of the computation the space-fixed (x, y, z) and body fixed (x^*, y^*, z^*) coordinate are displaced with respect to each other. Secondly, the body-induced pressure contains a term proportional to forward speed.

$$p_{BD} = -\rho \, \frac{d\phi_{BD}}{dt} + \rho U \, \frac{\partial \phi_{BD}}{\partial x}$$

It is assumed that the general form for the force and moment vector, $F_k(t)$ generated by a velocity of mode j and time history $v_j(t)$ in the ship fixed reference system,

$$\vec{v}* \cdot \vec{n} = v_j(t) \, n_j(\vec{x}) \, ,$$

is

$$F_{kj}(t) = F_{kj(IMP)} \, \dot{v}(t) + \int_{-\infty}^{t} K_{kj}(t-\tau) \, v_j(\tau) \, d\tau$$

where the first term represents virtual moss effects and the second term contains the memory effects resulting from the free surface. If, for example, a step function is imposed in mode j,

$$\dot{v}_j = H(t)$$

then the solution is

$$\tilde{F}_{kj}(t) = \int_{0}^{t} K(\tau) d\tau \qquad t > 0$$

and $\tilde{F}_{kj(IMP)}$ is defined by

$$\tilde{F}_{kj(IMP)} = -\rho \iint_{S_B} dS_B \, \tilde{\phi}_j(\vec{x}, \, 0+) \, n_k(\vec{x}) \, .$$

From integration by parts for

$$\vec{v}* \cdot \vec{n} = v_j \, n_j(\vec{x}) \, \cos\omega t$$

the solution is

$$F_{kj}(t) = \omega \left[\tilde{F}_{kj(IMP)} \, \sin\omega t + \int_{0+}^{\infty} \tilde{F}_{kj}(\tau) \, \sin\omega(t-\tau) \right]$$

$$= \omega\sin\omega t \left[\tilde{F}_{kj(IMP)} + \int_{0+}^{\infty} \tilde{F}_{kj}(\tau) \, \cos\omega\tau \, d\tau \right]$$

$$- \omega\cos\omega t \int_{0+}^{\infty} F_{kj}(\tau) \, \sin\omega\tau \, d\tau$$

The first term in the expression for F_{kj} is in phase with the acceleration and may be thought of as a generalized added mass term. The second term, in phase with velocity, represents a

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damping. The hydrostatic effect, not included in these dynamic forces, would, of course, give a term proportional to displacement.

Wave excited forces for harmonic waves can be computed using similar techniques. Consider an ambient wave field defined by

$$\phi = \text{Re} \left\{ A \, e^{ik(x \cos\theta + y \sin\theta) - kz - i\omega t} \right\}$$

$$= \text{Re} \left\{ A \, e^{ik(x^* \cos\theta + y^* \sin\theta - kz^* - i\omega_e t} \right\}$$

where A is the complex amplitude and ω_e is the encounter frequency defined as

$$\omega_e \equiv \omega - Uk \cos\theta.$$

The resulting ambient velocity and acceleration field can be written in the form

$$\vec{v}(\vec{x}^*, t) = \vec{v}_1(\vec{x}^*) \sin \omega_e t + \vec{v}_2(\vec{x}^*) \cos \omega_e t$$

and

$$\vec{a}(\vec{x}^*, t) = \omega_e \, \vec{v}_1(\vec{x}^*) \cos \omega_e t - \omega_e \, \vec{v}_2(\vec{x}^*) \sin \omega_e t$$

$$= a_1(t) \, \vec{v}_1(x^*) + a_2(t) \, v_2(\vec{x}^*)$$

where

$$\vec{v}_1(x) = I_m \left\{ A \, \vec{\nabla} \cdot e^{ik(x^* \cos\theta + y^* \sin\theta) - kz^*} \right\}$$

and

$$\vec{v}_2(x) = \text{Re} \left\{ A \, \vec{\nabla} \cdot e^{ik(x^* \cos\theta + y^* \sin\theta) - kz^*} \right\}$$

Now assume the time-domain method of solution is applied to the following pair of time step problems. In each case the hull and the surrounding fluid is at rest for $t < 0$. After $t = 0$; the body boundary conditions are

$$\vec{v}^* \cdot \vec{n} = \vec{v}_1(\vec{x}^*) \cdot \vec{n}(\vec{x}^*) \, a_1(t) \qquad t \geq 0$$

$$= \vec{v}_1(\vec{x}^*) \, n(\vec{x}^*) \, H(t), \text{ all } t$$

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for the first case and

$$\vec{v}* \cdot \vec{n} = \vec{v}_2(\vec{x}*) \cdot \vec{n}(\vec{x}*) \, a_2(t) \qquad t \geq 0$$

$$= \vec{v}_2(\vec{x}*) \cdot \vec{n}(\vec{x}) \, H(t), \text{ all } t.$$

i.e.,
$$a_1(t) = a_2(t) = H(t)$$

for the second case. The pair of time-domain forces and moments
for each of the six degrees of freedom obtained by applying the
time-domain computational solution to these two step problems for
arbitrary $a_1(t)$ and $a_2(t)$

$$F_{k1}(t) = \tilde{F}_{k1(IMP)} \, a_1(t) + \int_0^t \tilde{F}_{k1}(\tau) \, a_1(t-\tau) d\tau$$

$$F_{k2}(t) = \tilde{F}_{k2(IMP)} \, a_2(t) + \int_0^t \tilde{F}_{k2}(\tau) \, a_2(t-\tau) d\tau$$

Where the pair of time-domain forces and moments for each of the
six degrees of freedom obtained by applying the time-domain
computational method to these two problems are written as

$$\tilde{F}_{k1}(t) \text{ and } \tilde{F}_{k2}(t),$$

respectively where the tilda ~ symbol indicates a time-domain
solution, the subscript k refers to the degree of freedom of the
force or moment, and the remaining subscript indicates which of
the pair of body boundary conditions were employed.

The forces and moments generated by the harmonic velocity
field

$$\vec{v}(\vec{x}*, \ t) = \vec{v}_1(\vec{x}*) \sin \omega_e t + \vec{v}_2(\vec{x}*) \cos \omega_e t$$

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may be expressed in terms of the time-domain solutions as

$$F_k = \omega_e \int_0^\infty \tilde{F}_{k1}(\tau) \cos \omega_e(t-\tau)\, d\tau - \omega_e \int_0^\infty \tilde{F}_{k2}(\tau) \sin \omega_e(t-\tau)\, d\tau$$

$$F_k = \omega_e \cos\omega_e t \left[\tilde{F}_{k1(IMP)} + \int_{0+}^\infty [\tilde{F}_{k1}(\tau) \cos \omega_e\tau + \tilde{F}_{k2}(\tau) \sin \omega_e\tau]d\tau \right]$$

$$-\omega_e \sin\omega_e t \left[\tilde{F}_{k2(IMP)} + \int_{0+}^\infty [\tilde{F}_{k2}(\tau) \cos \omega_e\tau - \tilde{F}_{k1}(\tau) \sin \omega_e\tau]d\tau \right]$$

where

$$\tilde{F}_{k1(IMP)} \quad -\rho\gamma \iint_{S_B} ds\ \breve{\phi}_1(\vec{x}*,\ t_o)\ n_k(\vec{x}*)$$

and

$$\tilde{F}_{k2(IMP)} = -\rho\gamma \iint_{S_B} ds\ \breve{\phi}_2(\vec{x}*,\ t_o)\ n_k(\vec{x}*)$$

Heave Motion of a Floating Sphere

As a simple example of the method described above, the computer program capable of handling any three dimensional hull shape was applied to the case of heave motion for a semi-submerged sphere, i.e., a hemispherical hull. The computer program is completely general and makes no assumptions concerning either lateral or longitudinal symmetry of the body or the flow. For the case of zero forward speed, however, the symmetry of the hemispherical hull requires that only heave force result from heave motion. The magnitudes of other forces calculated numerically should be one measure of the adequacy of the surface panels chosen to represent the body and the program itself.

In the test case 60 panels were used to represent the hemisphere with spacings specified by

$$\Delta\psi = 2\pi/12 \text{ and } \Delta\theta = \pi/5$$

where $0 \leq \psi < 2\pi$ is the circumferential angle and $0 \leq \theta \leq \pi$ is the polar angle. The hull-fixed coordinates are specified at each of the four corners of each panel with coordinates given by

$$x^* = a \cos\psi \cos\theta$$

$$y^* = a \sin\psi \cos\theta$$

and
$$z^* = a \sin\theta$$

with radius a set to unity in this test case.

If the hull is not flat over a panel, its finite size causes errors in the computation of the panel center and panel area. For simplicity the panel center is computed to be at the mean of the four corner points. For a convex hull such as the hemisphere these points fall slightly inside the physical hull rather than directly on it. More important is the error in the computed hull area. Again for simplicity, the pair of diagonals,

$$\vec{x}_{31} = \vec{x}_3 - \vec{x}_1$$

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and

$$\vec{x}_{42} = \vec{x}_4 - \vec{x}_2$$

are used to estimate the panel area,

$$A = \frac{1}{2} |\vec{x}_{42} \times \vec{x}_{31}|$$

and unit normal,

$$\vec{n} = \vec{x}_{42} \times \vec{x}_{31}/|\vec{x}_{42} \times \vec{x}_{31}|$$

where x_1, x_2, x_3 and x_4 are the coordinates of the four corner points of the panel in the body-fixed system. Although it is immaterial whether any single panel normal points inward or outward, for simplicity all normals are assumed to be outward in the description of the method and in the sample problem. In the case of convex bodies the areas are underpredicted by an amount which depends on the panel size. Likewise, the areas are also underestimated for concave portions of the physical hull.

For the case of a hemispherical hull represented by sixty plane quadralateral panels the impulsive forces and moments resulting from an instantaneous change in the heave (z) velocity (+ downward) from zero at t = 0- to unity at t = 0+, was computed to be (for unit density, $\rho = 1$):

$$\tilde{F}_{13(IMP)} = \text{surge force} = -0.00003$$

$$\tilde{F}_{23(IMP)} = \text{sway force} = .00656$$

$$\tilde{F}_{33(IMP)} = \text{heave force} = -1.00150 \quad .$$

$$\tilde{F}_{45(IMP)} = \text{roll moment} = .00004$$

$$\tilde{F}_{53(IMP)} = \text{pitch moment} = -.00011$$

and $\quad \tilde{F}_{63(IMP)} = \text{yaw moment} = -.00012 \quad .$

From theory

$$\tilde{F}_{k3(IMP)} = 0 \quad k \neq 3$$

and

$$\tilde{F}_{33(IMP)} = -\pi/3 = -1.04720 \quad .$$

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The computed terms are thus $\leq O(10^{-3})$ for all forces and moments other than heave. The computed heave force is about 3 percent too low; a number which could be reduced with a finer hull resolution in the panels.

The time-domain numerical method was applied to the semi-submerged sphere. Initially (t < 0) the flow and the body was specified to be at rest. At time t = 0, a step function was applied to the heave velocity. Forward speed was set at zero. Thus the hull velocity boundary condition is

$$(\vec{v}_{BD}(\vec{x}*) + \vec{v}_{FS}(\vec{x}*)) \cdot \vec{n}(\vec{x}*) = n_z(\vec{x}*) \, H(t)$$

where $\vec{x}*$ is any point on the hull surface and $H(t)$ is the unit step function. For the initial impulse time step of infinitesmal duration starting at $t = 0$, a velocity boundary condition was applied at each panel center as

$$\sum_{j=1}^{N_B} E_{ij} \, \sigma_j(t = 0+) = \vec{v}_{BD_i} \cdot \vec{n}_i = n_{zi}$$

(since $\vec{v}_{FS_i}$ is zero at t = 0+). The source strengths at t = 0+, $\sigma_j(t = 0+)$ were then used to find the potentials on the panels and, therefore, the forces and moments acting on the body in all six degrees of freedom as a result of the initial impulsive step from zero to unity in heave velocity (the third mode). That is equivalent to

$$\tilde{F}_{kj(IMP)} \qquad k = 1, \, 2, \, \ldots \, 6 \, .$$

In practice these impulsive forces are equivalent to the formula given earlier,

$$\tilde{F}_{kj(IMP)} = -\rho \iint_{S_B^*} dS_B \, \tilde{\phi}_j(\vec{x}*, \, 0+) \, n_k(\vec{x}*)$$

For later time steps the time-domain force

$$\tilde{F}_{kj}(t) = \int_{0+}^{t} K_{kj}(\tau) d\tau, \qquad t > 0+$$

was computed using the method presented in this report with an acceleration body boundary condition of the form

$$\sum_{j=1}^{N_B} E_{ij} \, \dot{\sigma}_j(t) = \alpha_i^*(t) - a_{ps} \cdot n_i - U \frac{\partial}{\partial x} \vec{v}_{xs} \qquad t > 0+$$

(with the forward speed U set to be zero)

since the prescribed normal acceleration a_i^* is identically zero for $t > 0+$. The resulting values for $\dot{\sigma}_i(t)$, i.e., the time derivative of the source strength density on each panel, gives the time rate of change of the body potential and, therefore, the body-induced pressure p_{BD_i} at each panel. By adding the free-surface induced pressures, the total pressure acting on each panel was determined. This produced the net computed force

$$\tilde{F}_{k3}(t)$$

acting on the body for $t > 0+$.

For the numerical test case, the forces and moments of the class $\tilde{F}_{k3}(t)$, $k \neq 3$ were all insignificant. Figure 1 shows $\tilde{F}_{33}(t)$ computed under the following numerical conditions -

$$L_x = \text{maximum length scale in } x = 2.5$$

$$\ell_x = \text{minimum length scale in } x = 0.25$$

$$L_y = \text{maximum length scale in } y = 2.5$$

$$\ell_y = \text{minimum length scale in } y = 0.25$$

T = maximum time scale $\quad = 8.0$

g = acceleration of gravity $\quad = 1.0$

ρ = fluid density $\quad\quad\quad\quad = 1.0$

a = radius of sphere $\quad\quad\quad = 1.0$

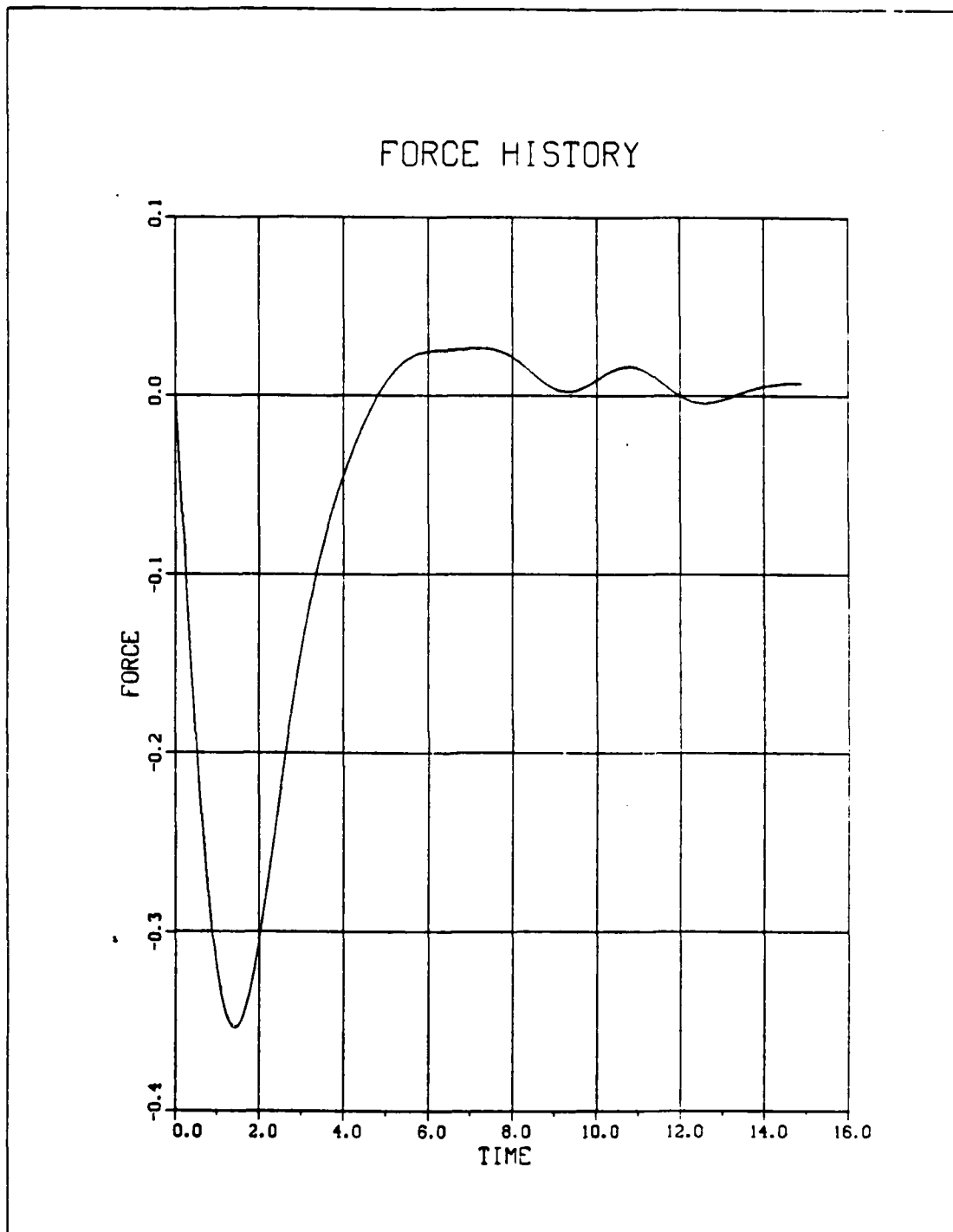Δt = time step size $\quad\quad\quad = 0.10$

Figure 1. Time history of force history induced
by a unit step in heave velocity start-
ing at t=0 for a semisubmerged sphere.

The first five of these conditions produce 17 wave lengths for
the free surface representation in both the x and y directions or
17 x 17 modes. For the harmonic case

$$v_3(t) = \cos\omega t \ ,$$

the force acting in the z direction (+ downward) is given by

$$F_3(t) = -\omega \sin(\omega t) \left[\tilde{F}_{33(IMP)} + \int_{0+}^{\infty} \tilde{F}_{33}(\tau) \cos(\omega\tau)d\tau \right]$$

$$+\omega \cos(\omega t) \int_{0+}^{\infty} \tilde{F}_{33}(\tau) \sin(\omega\tau)d\tau \ .$$

This formula can be evaluated numerically and compared with the
classical results of Havelock [3].

Havelock expresses the z force induced by the heave
oscillations

$$v_3 = \cos\omega t$$

as

$$Z = \frac{2}{3} \pi\rho a^3 k \sin\omega t - \frac{2}{3} \pi\rho a^3 2h \ \omega \cos\omega t$$

where a is the radius and ρ is the density (both are unity). The
coefficients k (not a wave number) and 2h determine the added mass
and hydrodynamic damping of the sphere in heave. These two coef-
ficients are commonly expressed in terms of the nondimensional
variable $\omega^2 a/g$.

These coefficients defined by Havelock are related to
the values derived from the time-domain solution for a step func-
tion in the heave mode as

$$k(\omega^2 a/g) = -\frac{3}{2\pi} \tilde{F}_{33(IMP)} + \int_{0+}^{\infty} \tilde{F}_{33}(\tau) \cos(\omega\tau)d\tau$$

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and

$$2h(\omega^2 a/g) = - \frac{3}{2\pi} \int\limits_{0+}^{\infty} \tilde{F}_{33}(\tau) \sin(\omega\tau)d\tau$$

Figure 2 shows coefficients $k(\omega^2 a/g)$ and $2h(\omega^2 a/g)$ calculated in this manner using $\tilde{F}_{33(IMP)} = -1.0015$ and $\tilde{F}_{33}(\tau)$ as shown in Figure 1. The resulting curves, close to those predicted by Havelock using analytic methods, given in reference [3]. Tabular results for this problem were also given by Kim [4]. Table 1 below shows a comparison between Kim's results and the computed results for various values of $\omega^2 a/g$.

Table 1. Comparison of analytic and numerical
results for added mass and damping
coefficients of a semisubmerged sphere
of radius a oscillating with frequency ω.

Added Mass Coefficients

$\omega^2 a/g$	$k(\omega^2 a/g)$ [Ref. [4]]	$k(\omega^2 a/g)$ [Computed]
0.50	0.60	0.58
1.00	0.43	0.43
1.50	0.39	0.39
2.00	0.39	0.39
2.50	0.40	0.39
3.00	0.41	0.38

Damping Coefficients

$\omega^2 a/g$	$2h(\omega^2 a/g)$ [Ref. [4]]	$2h(\omega^2 a/g)$ [Computed]
0.50	0.34	0.33
1.00	0.25	0.25
1.50	0.17	0.17
2.00	0.11	0.12
2.50	0.07	0.10
3.00	0.06	0.06

MASS AND DAMPING COEFFICIENTS

$k(\omega^2 a/g)$
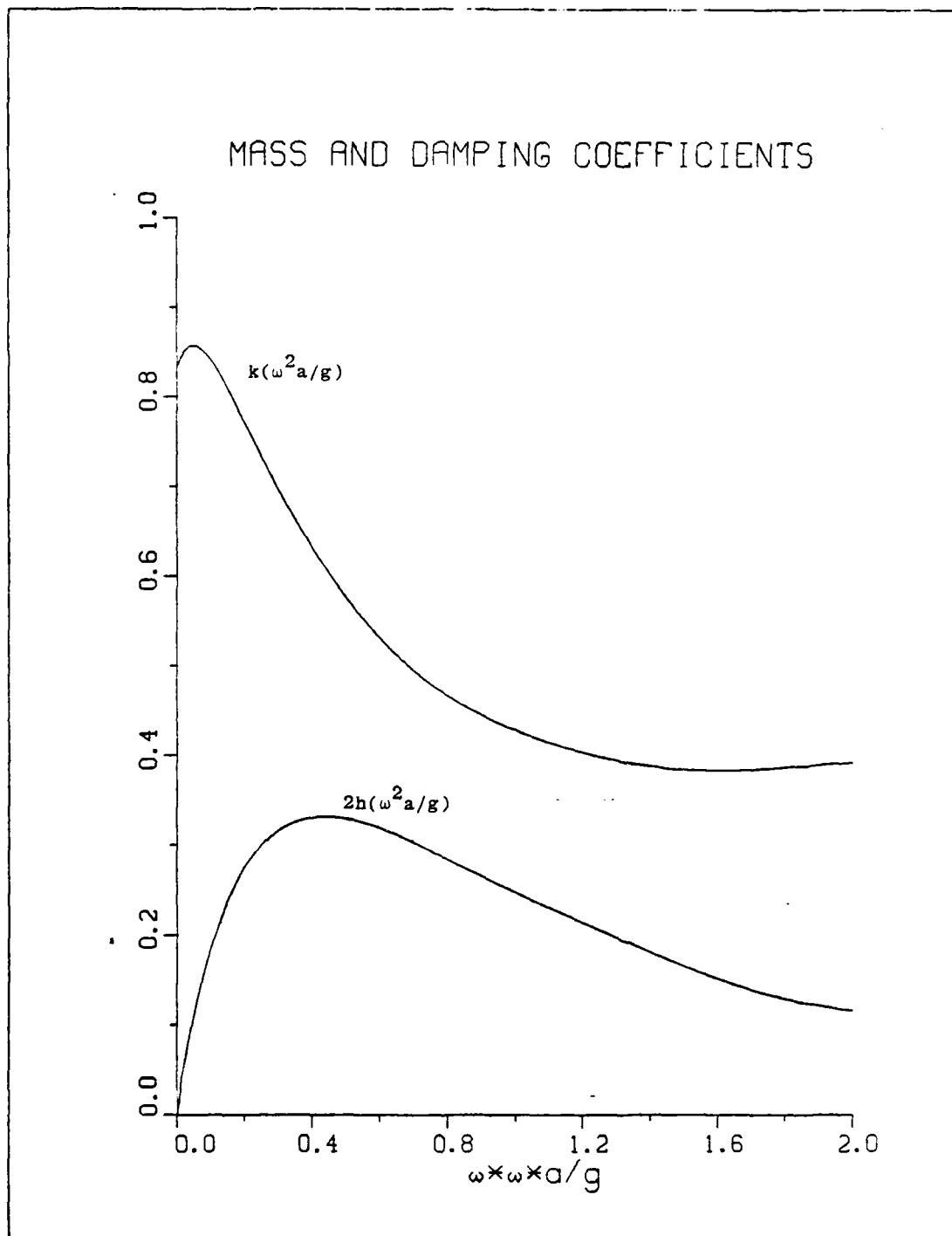
$2h(\omega^2 a/g)$

$\omega * \omega * a/g$

Figure 2. Added mass and damping coefficients
from computation. Corresponds to
Figure 1 in Reference [3].

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